This question paper contains 7 printed pages]

Your Roll No................

6602

B.Sc.(Hons.) Computer Science/I Sem. B

Paper CSHT-102 : Discrete Structures

(Admissions of 2011 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* the questions.

Parts of a question must be performed together.

Use of Scientific Calculator is allowed.

1. (*a*) A TV survey shows that 60 percent people see program A, 50% see program B, 50% see program C, 30% see program A and B, 20% see program B and C, 30% see program A and C and 10% do not see any program. Find :

(*i*) What % see program A, B and C ?

(*ii*) What % see program A only ? 4

P.T.O.

(*b*) Show that any integer composed of 3^n identical digits is divisible by 3^n using Mathematical Induction. 4

(*c*) Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from R to R. 4

2. (*a*) Show that the relation $\leq$ (less than or equal to) defined on the set of positive integers is a partial order relation. 3

(*b*) Let a be a numeric function such that :

$$a_r = \begin{cases} 2 & 0 \leq r \leq 3 \\ 2^{-r} + 5 & r \geq 4 \end{cases}$$

Determine ∇a and Δa. 4

(*c*) Solve the recurrence relation

$$a_{n+2} - 3a_{n+1} + 2a_n = 0$$

by the generating function method with initial conditions $a_0 = 2$ and $a_1 = 3$. 5

(*d*) Use Master method to give tight asymptotic bounds for the following Recurrence relation

$$T(n) = 4T(n/2) + n^3.$$ 2

3. (*a*) Show the equivalence

$$\neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p.$$ 3

(*b*) Prove the conclusion from the given sets of premises

$$P \rightarrow (Q \rightarrow R),\ Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S).$$ 5

(*c*) Translate these statements into English.

Let $P(x, y)$ = 'x has sent a letter to y', where universe of discourse of both x and y consists of all students in a class.

(*i*) $\exists y\ \exists x\ P(x, y)$

(*ii*) $\forall x\ \exists y\ P(x, y)$. 4

(*d*) Verify that the proposition $p \vee \urcorner (p \wedge q)$ is a tautology. 3

4. (*a*) Evaluate the sum 3

$$\sum_{k=1}^{\infty} (2k + 1) x^{2k}.$$

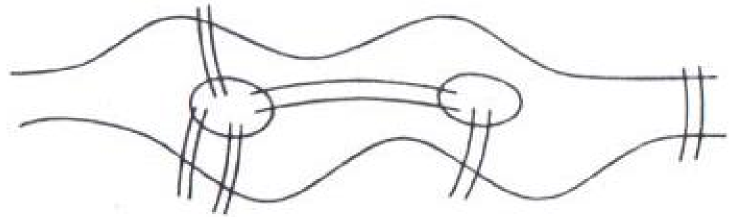
(*b*) How many different ways are there to select 4 different players from 10 players on a team to play four tennis matches, where the matches are ordered. 3

(*c*) Show that among any group of five integers, there are at least two integers with the same remainder when divided by 4. 3

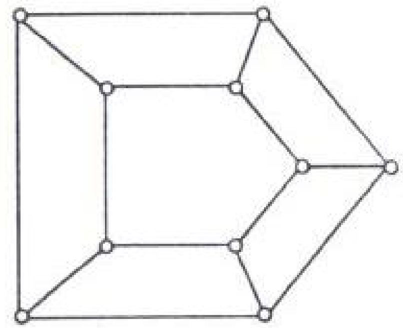
5. (*a*) Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane ? 2

(*b*) How many vertices does a full 5-ary tree with 100 internal vertices have ? 2

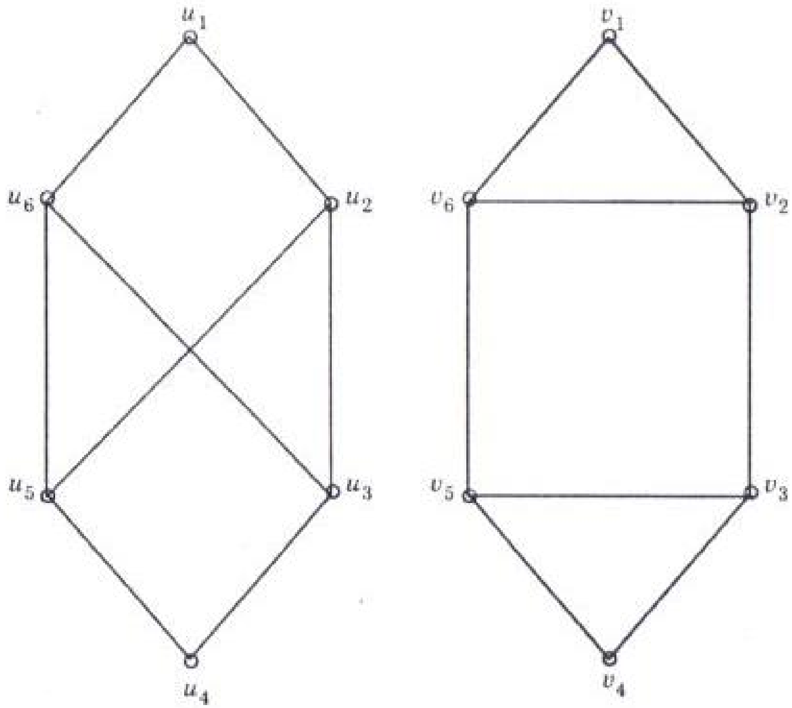
(*c*) Can someone cross all the bridges shown in this map exactly once and return to the starting point ? If so, determine the path ? 4



(*d*) Derive an expression for the chromatic number of C_{nn} where $n > 3$. C_{nn} is a graph with two concentric cycles and n vertices, connected as shown below : 3



(*e*) Determine whether G_1 and G_2 are isomorphic or not ? 4



G_1 G_2

6. (*a*) Suppose that the no. of bacteria in a colony triples every hour. 3

(*i*) Set up a recurrence relation for the number of bacteria after *n* hours have elapsed.

(*ii*) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours ?

(*b*) Show that :

$$x^2 + 4x + 17$$

is $O(x^3 - 2x^2 - 5)$. 4

(*c*) Show that if

$f(n) = O(g(n))$ and $f(n) = \Omega\ (g(n))$ then

$f(n) = \Theta\ ((g(n))$. 3